Buckling and Postbuckling Behavior of Shallow Shells

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Theme

THE advent of lighter weight, higher strength materials, and optimization has introduced thin walled structures in which buckling precedes failure. Hence stability has become one of the main governing factors in the design of shell panels. Archer[1] et al. have found the buckling loads of shallow spherical shells adopting finite-difference method. Leicester[2] using double series, and Dhatt[3] through finite element procedure, have investigated the buckling and postbuckling behavior of spherical shells. In the present analysis a matrix method is used to study the buckling and postbuckling behavior of shallow shells due to uniformly distributed transverse loads. Numerical work has been done for cylindrical and spherical shells with rectangular planform. Both the clamped and the simply supported boundary conditions have been treated.

Content

The equation of the doubly curved shell surface can be given as $z=z(x,y)$. As the shell is shallow, the curvatures k_x, k_y, and k_{xy} are assumed such that $k_x=z,_{xx}$, $k_y=z,_{yy}$, and $k_{xy}=z,_{xy}$. Using Vlasov's theory, the 3 equations of equilibrium for shallow shells of constant curvature can be written (vide Ref. 1) in terms of u, v, w the displacements of the middle surface in the x,y,z directions. Let h be the thickness of the shell; ν, Poisson's ratio; q, the transverse load per unit area, and $D=Eh^3/[12(1-\nu^2)]$, where E is Young's modulus of elasticity. The equations of equilibrium are nondimensionalized by putting $\bar{x}=x/a$, $\bar{y}=y/b$, $\bar{w}=w/h$, $\bar{u}=au/h^2$, $\bar{v}=bv/h^2$, $\bar{z}=z/h$, $c=a/b$, $\bar{k}_{\bar{x}}=a^2k_x/h$, $\bar{k}_{\bar{y}}=b^2k_y/h$, $\bar{k}_{\bar{x}\bar{y}}=ab\ k_{xy}/h$,and $\bar{P}=qa^4/(12\ Dh)$, where a and b are the sides of the shell in the x and y directions. The boundary conditions adopted are

a) Clamped at $\bar{x}=0,1;\quad \bar{w}=\bar{w},_{\bar{x}}=\bar{u}=\bar{v}=0$

at $\bar{y}=0,1;\quad \bar{w}=\bar{w},_{\bar{y}}=\bar{u}=\bar{v}=0$

b) Simply supported at $\bar{x}=0,1;\quad \bar{w}=\bar{w},_{\bar{x}\bar{x}}=\bar{u}=\bar{v}=0$

at $\bar{y}=0,1;\quad \bar{w}=\bar{w},_{\bar{y}\bar{y}}=\bar{u}=\bar{v}=0$

The nondimensionalized shallow shell equations are solved by a matrix method similar to the one used by Hadid[4] for the linear analysis of conoidal shell. In this method the shallow shell is divided by a set of vertical planes parallel to the edges of the shell (Fig. 1). Taking the highest derivatives of displacements as unknowns, the governing equations are transformed into nonlinear algebraic equations, and they are solved using the Newton Raphson method. The procedure followed for the generation of the gradient matrix is akin to the one adopted by Chu[5] et al. In this method the boundary conditions are satisfied exactly at discrete points.

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Clamped shallow circular cylindrical panels with $\bar{k}_{\bar{x}}=15,32,45$ ($a=b=20''$, $h=0.125''$, $E=450{,}000$ psi, $\nu=0.3$) were analyzed and the results are shown in Fig. 2. The load deflection curve for $\bar{k}_{\bar{x}}=32$ has been reported by Dhatt.[6] Although the results obtained in the present analysis using 5×5 and 7×7 meshes (25 and 49 internal points for complete shell) diverge from the true value for large displacements, the values of the 9×9 mesh agree well with those of Dhatt.[6] This is due to the fact that, with a smaller number of mesh points it is not possible for the solution to take the complicated shape the shell assumes after buckling. This is also evident for other curvatures. As the curvature is increased a finer mesh is required to trace the load deflection path correctly in the post-

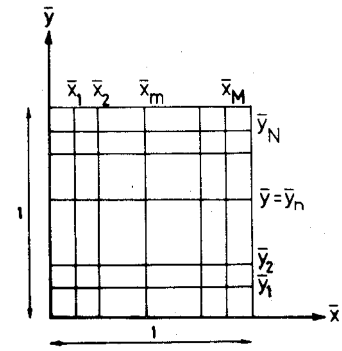

Fig. 1 Subdivision of shell.

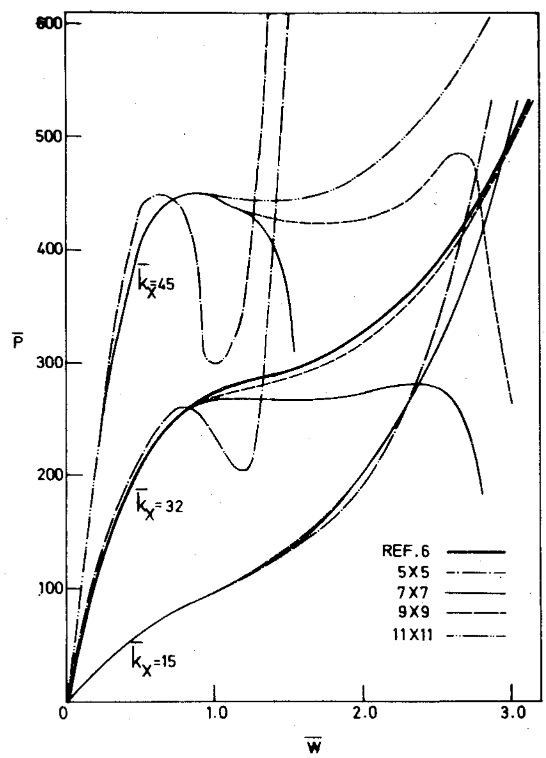

Fig. 2 Central deflection curve for clamped circular cylindrical panel.

Table 1 Snap buckling loads of spherical shell

	λ^2	Values of $\bar{Q}$ Authors 5×5	7×7	9×9	11×11	Archer 5×5	8×8	10×10	12×12
Clamped	9	.6485	....	....	....		.635[a]	....	....
	10	.6091	.6177	.6242				...	.600
	16	.5730	.5700	.5678			.625	....	.575
	36	.8265	.7262	.7427	.7438		.863	.813	.775
	64	.6383	.7099	.7020	.7139		.988	.850	.800
	100	.7820	....	.7510	.7190		1.150	....	.875
Simply Supported	36	.5836	.5936	.6145		1.000	.738	.675	.650

[a] Read from Fig. 6 in Ref. 1.

buckling range. One interesting feature noted in Fig. 2 is that although finer meshes are required to trace the solution path in the postbuckling range, even the coarsest mesh gives the snap load close to the exact value.

Convergence of snap loads $\bar{Q}(=qR^2[12(1-\nu^2)]^{1/2}/(4Eh^2))$ for clamped spherical shell on square base were studied using different mesh sizes and for values of $\lambda^2=9$, 10 16, 36, 64, and 100 ($\lambda^2=[12(1-\nu^2)]^{1/2}a^2/4hR$, where R is the radius of the spherical shell). The results for these cases are presented in Table 1 and are compared with those of Archer.[1] It can be seen that the results for a 7×7 mesh are comparable with those of the converged values. This trend can be observed even for large values of λ^2. The values of snap loads for a simply supported shell with $\lambda^2=36$ are also given in Table 1.

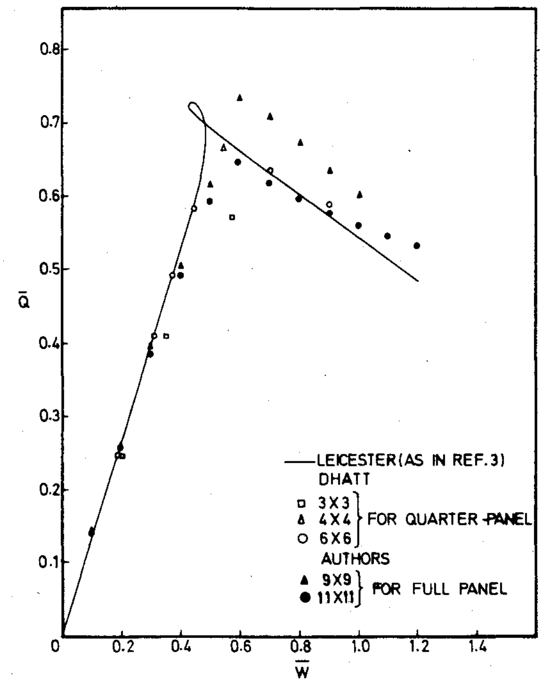

Fig. 3 Central deflection curve for simply supported spherical shell.

Results have been obtained for a simply supported shell ($a=b=43.69''$, $R=100''$, $h=1''$, $E=10{,}000$ psi, $\nu=0.3$) using 9×9, and 11×11 meshes (75 and 108 unknowns, respectively, for quarter shell). For this case Dhatt[3] has reported values using the finite element method (441 unknowns for quarter shell) and compared with those of Leicester. The results obtained in the present analysis are compared with those of Dhatt and Leicester in Fig. 3. It can be seen that there is good agreement with those of Dhatt although much less number of unknowns are involved in the present analysis.

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